9. Contextual Problems

1

A scientist grows a certain type of bacteria in a laboratory. The table shows the number of bacteria measured at different times of the day.

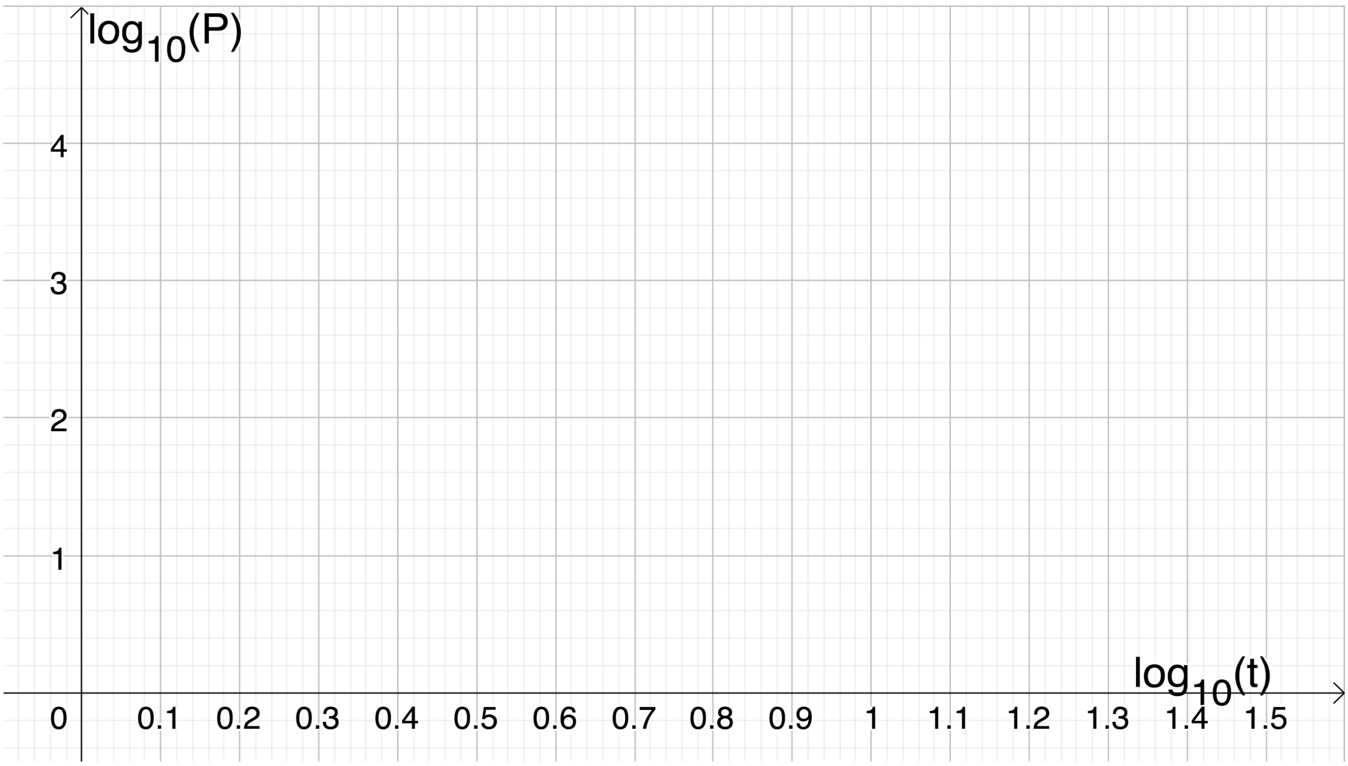
Time	8:00am	10:00am	1:00pm	3:00pm	8:00pm	10:00pm
Number of bacteria, P	20	86	283	481	1219	1511

This growth may be modelled by an equation of the form $P = at^b$, where P is the population of bacteria, t is the number of hours after 6:00am, and a and b are constants to be determined.

a. Show that, according to this model, the graph of $\log_{10} P$ against $\log_{10} t$ should be a straight line of gradient b. State, in terms of a, the intercept on the vertical axis.

b. Complete the table of values below, writing your answers to 2 decimal places, and plot $\log_{10} P$ against $\log_{10} t$. Draw by eye a line of best fit for the data.

t	2	4	7	9	14	16
P	20	86	283	481	1219	1511
$\log_{10} t$						
$\log_{10} P$						



c. Use your graph to find the equation for P in terms of t.

d. Estimate the number of bacteria at 5:00pm.

5

Arnold records the number of subscribers to his YouTube channel every week. The table below shows some of his records.

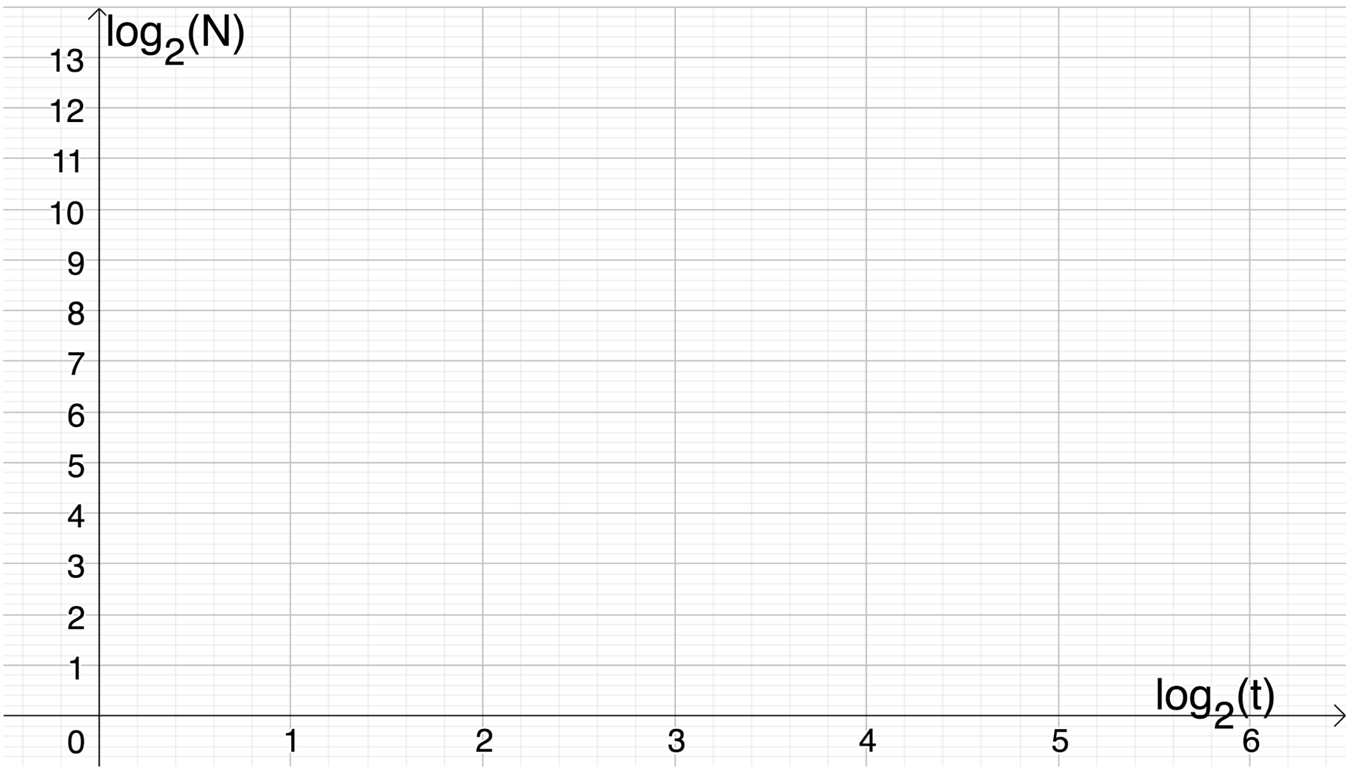
Week	Week 2	Week 5	Week 9	Week 15	Week 20	Week 25
Number of subscribers, N	14	107	378	1339	2724	5446

This growth may be modelled by an equation of the form $N = at^b$, where N is the number of subscribers, t is the number of weeks after the channel was first created, and a and b are constants to be determined.

a. Show that, according to this model, the graph of $\log_2 N$ against $\log_2 t$ should be a straight line of gradient b. State, in terms of a, the intercept on the vertical axis.

b. Complete the table of values below, writing your answers to 2 decimal places, and plot $\log_2 N$ against $\log_2 t$. Draw by eye a line of best fit for the data.

t	2	5	9	15	20	25
N	14	107	378	1339	2724	5446
$\log_2 t$						
$\log_2 N$						



c. Use your graph to find the equation for N in terms of t.

d. Predict in which week the number of subscribers will be 20000.

8

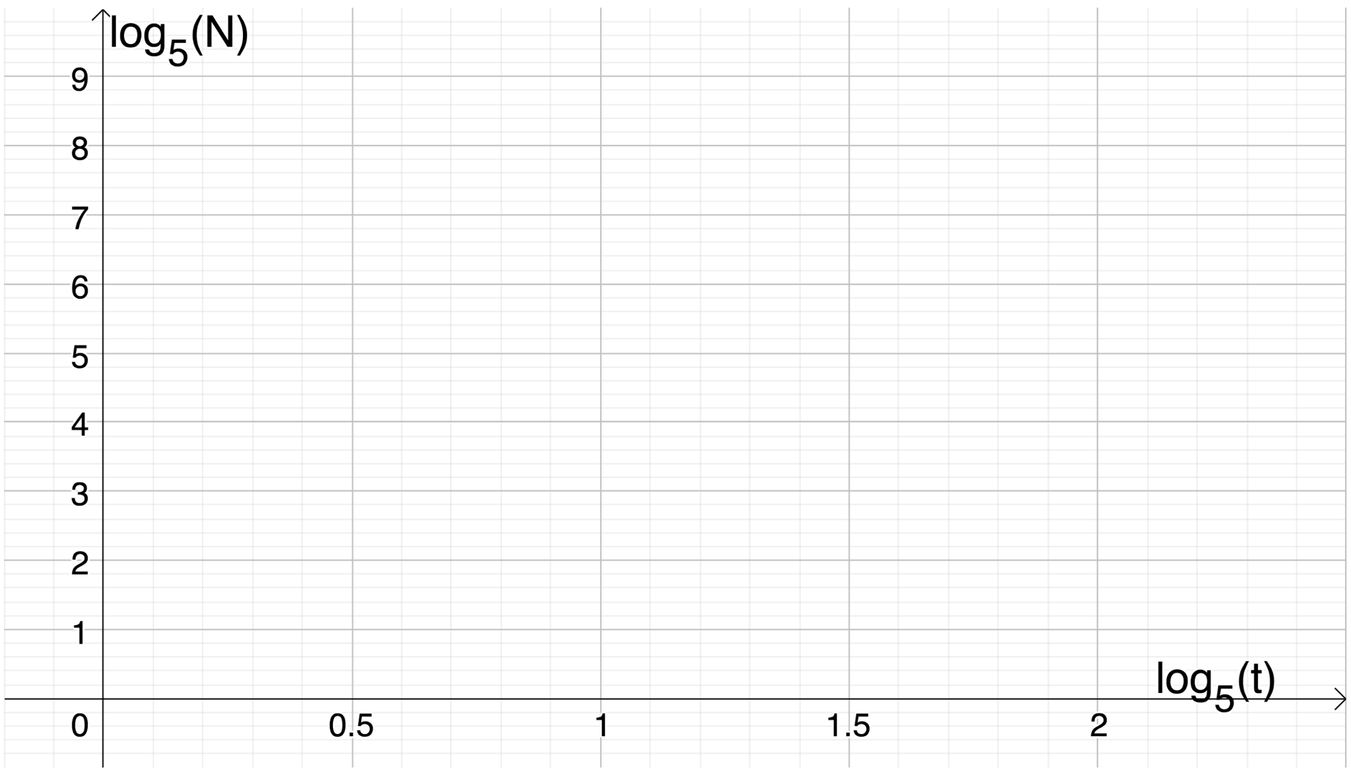
A UK monthly magazine was first published on January 1st 1999. The number of subscriptions fell significantly over the years between 2009 and 2019. The table below shows the number of subscriptions on January the 1st every two years from 2009 to 2019.

Year	2009	2011	2013	2015	2017	2019
Number, (N thousands)	119	23	14	11	7	5

This growth may be modelled by an equation of the form $N = at^b$, where N is the number of subscriptions in thousands, t is the number of years after 1999, and a and b are constants to be determined.

a. Complete the table of values below, writing your answers to 2 decimal places, and plot $\log_5 N$ against $\log_5 t$. Draw by eye a line of best fit for the data.

t	10	12	14	16	18	20
N	119	23	14	11	7	5
$\log_5 t$						
$\log_5 N$						



b. Use your graph to find the equation for N in terms of t.

c. Use your results to estimate how many subscriptions there were to the magazine on January 1st 2000. Explain whether this is likely to be a correct figure.

9

The weekly profits of a small café are recorded at the end of each week, and some of these are shown in the table below.

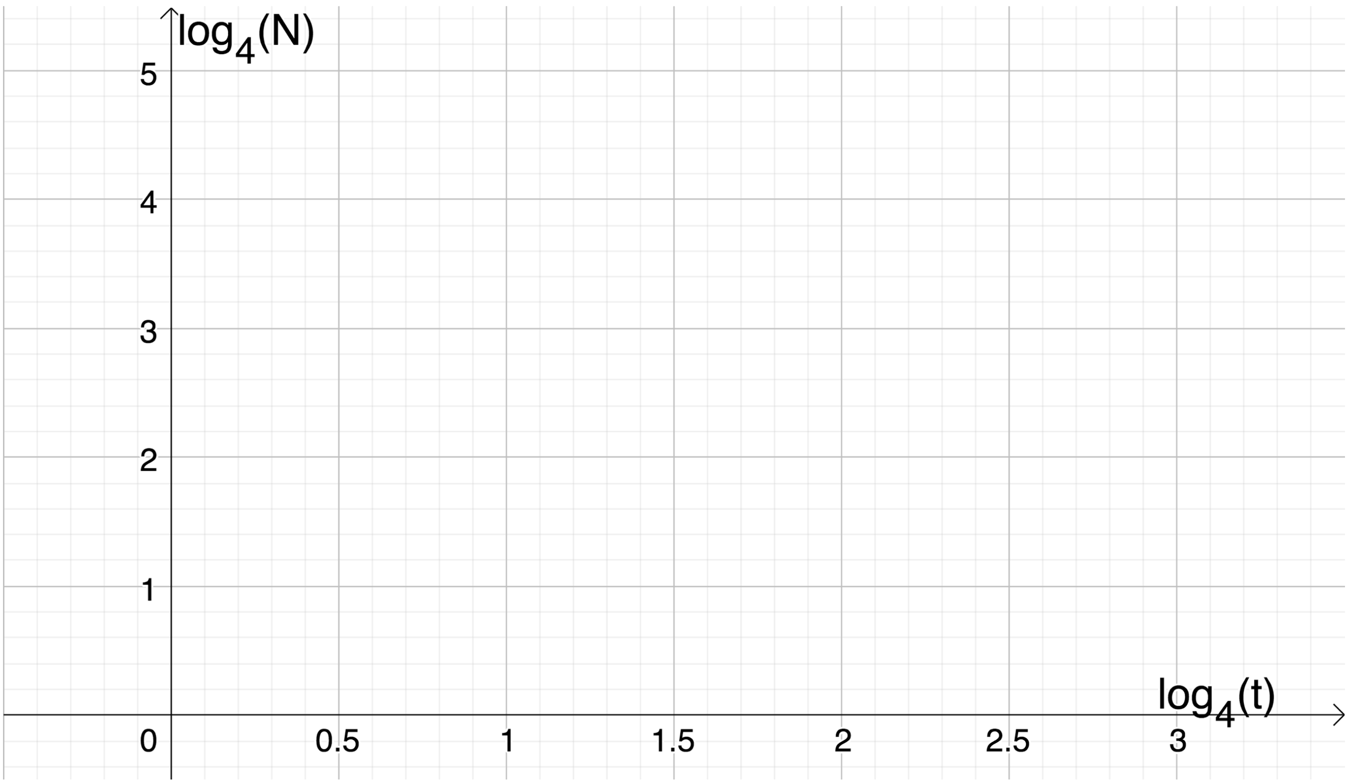
Week	4	8	12	16	20	24
Profit, (£P)	271	398	452	483	501	515

This growth may be modelled by an equation of the form $P = at^b$, where P is the profit in £, t is the number of weeks since opening, and a and b are constants to be determined.

a. Show that, according to this model, the graph of $\log_4 P$ against $\log_4 t$ should be a straight line of gradient b. State, in terms of a, the intercept on the vertical axis.

b. Complete the table of values below, writing your answers to 2 decimal places, and plot $\log_4 P$ against $\log_4 t$. Draw by eye a line of best fit for the data.

t	4	8	12	16	20	24
N	271	398	452	483	501	515
$\log_4 t$						
$\log_4 P$						



c. Use your graph to find the equation for N in terms of t.

10

The table below shows the inflation-adjusted price of gold per ounce every six months June 1977 to January 1980.

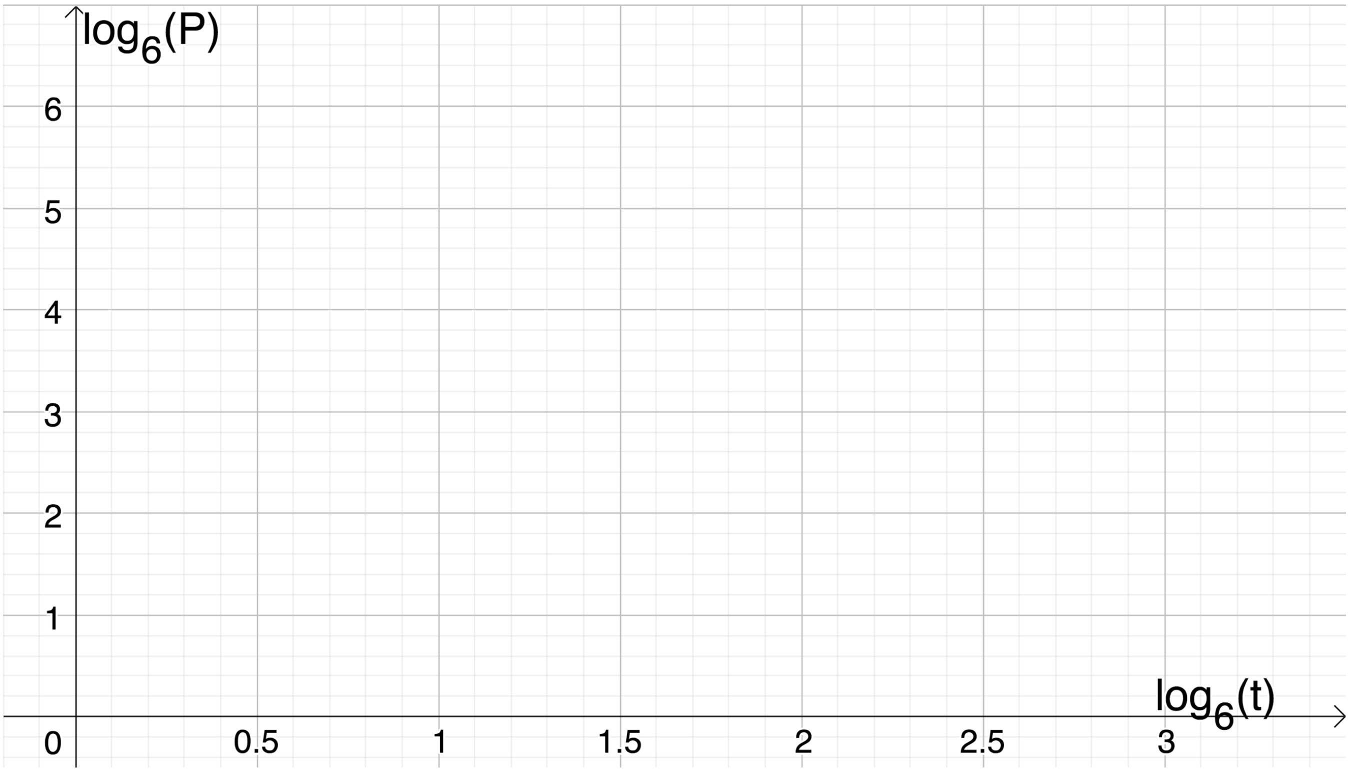
Year	Jun 1977	Jan 1978	Jun 1978	Jan 1979	Jun 1979	Jan 1980
Price, $\$P$	598.19	715.59	727.14	858.63	996.33	2248.15

This growth may be modelled by an equation of the form $P = at^b$, where $\$P$ is the price of gold per ounce, t is the number of months after January 1977, and a and b are constants to be determined.

a. Show that, according to this model, the graph of $\log_6 P$ against $\log_6 t$ should be a straight line of gradient b. State, in terms of a, the intercept on the vertical axis.

b. Complete the table of values below, writing your answers to 2 decimal places, and plot $\log_6 P$ against $\log_6 t$. Draw by eye a line of best fit for the data.

t	6	12	18	24	30	36
N	598.19	715.59	727.14	858.63	996.33	2248.15
$\log_6 t$						
$\log_6 P$						



c. Use your graph to find the equation for P in terms of t.

d. Use your answers to predict the price of gold in January 1982. The actual price of gold was $\$1050.49$. Find the percentage difference between your prediction and the actual value.

9. Contextual Problems

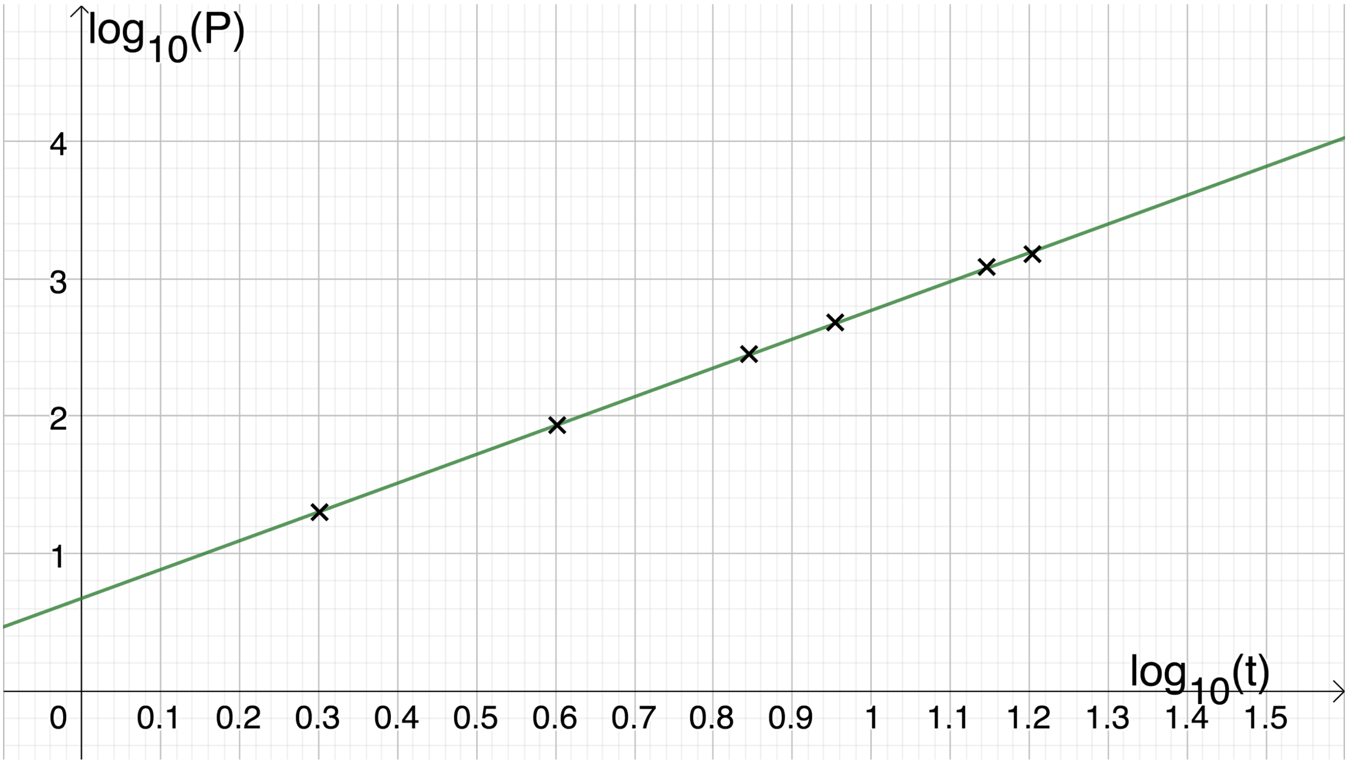
1

a. Show that, according to this model, the graph of $\log_{10} P$ against $\log_{10} t$ should be a straight line of gradient b. State, in terms of a, the intercept on the vertical axis.

$$P = at^b \Rightarrow \log_{10} P = \log_{10}(at^b)$$
$$\Rightarrow \log_{10} P = \log_{10} a + \log_{10} t^b$$
$$\Rightarrow \log_{10} P = \log_{10} a + b \log_{10} t$$

b. Complete the table of values below, writing your answers to 2 decimal places, and plot $\log_{10} P$ against $\log_{10} t$. Draw by eye a line of best fit for the data.

t	2	4	7	9	14	16
P	20	86	283	481	1219	1511
$\log_{10} t$	0.30103	0.60206	0.8451	0.95424	1.14613	1.20412
$\log_{10} P$	1.30103	1.9345	2.45179	2.68215	3.086	3.17926



c. Use your graph to find the equation for P in terms of t.

Gradient ≈ 2.095

Intercept ≈ 0.675

$$P = 4.727 \times t^{2.095}$$

d. Estimate the number of bacteria at 5:00pm.

$$P|_{t=11} \approx 718$$

5

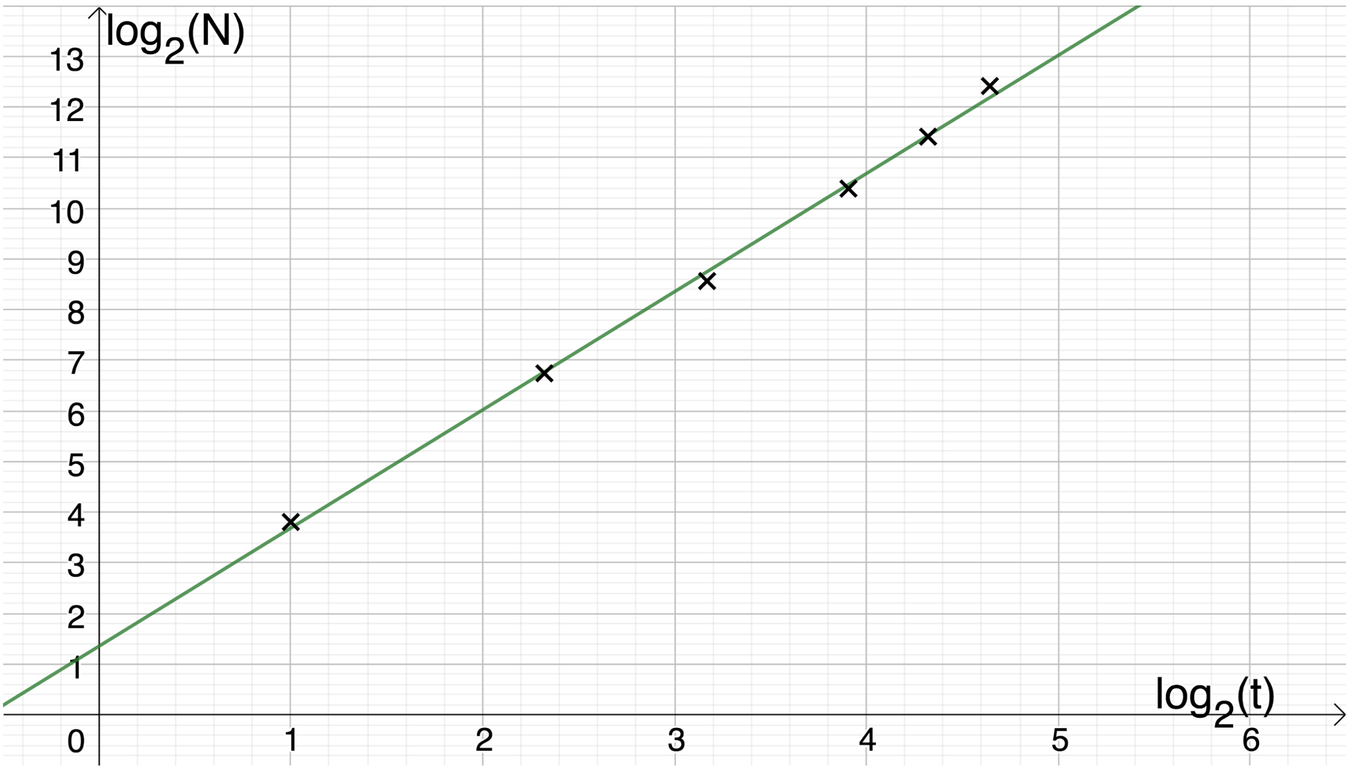
a. Show that, according to this model, the graph of $\log_2 N$ against $\log_2 t$ should be a straight line of gradient b. State, in terms of a, the intercept on the vertical axis.

$$N = at^b \Rightarrow \log_2 N = \log_2(at^b)$$
$$\Rightarrow \log_2 N = \log_2 a + \log_2 t^b$$
$$\Rightarrow \log_2 N = \log_2 a + b\log_2 t$$

Intercept on the vertical axis is $\log_2 a$

b. Complete the table of values below, writing your answers to 2 decimal places, and plot $\log_2 N$ against $\log_2 t$. Draw by eye a line of best fit for the data.

t	2	5	9	15	20	25
N	14	107	378	1339	2724	5446
$\log_2 t$	1	2.32193	3.16993	3.90689	4.32193	4.64386
$\log_2 N$	3.80735	6.74147	8.56224	10.38694	11.41151	12.41098



c. Use your graph to find the equation for N in terms of t.

$$N = 2.56 \times t^{2.333}$$

d. Predict in which week the number of subscribers will be 20000.

$$20000 = 2.56 \times t^{2.333} \Rightarrow t \approx 46.6$$

8

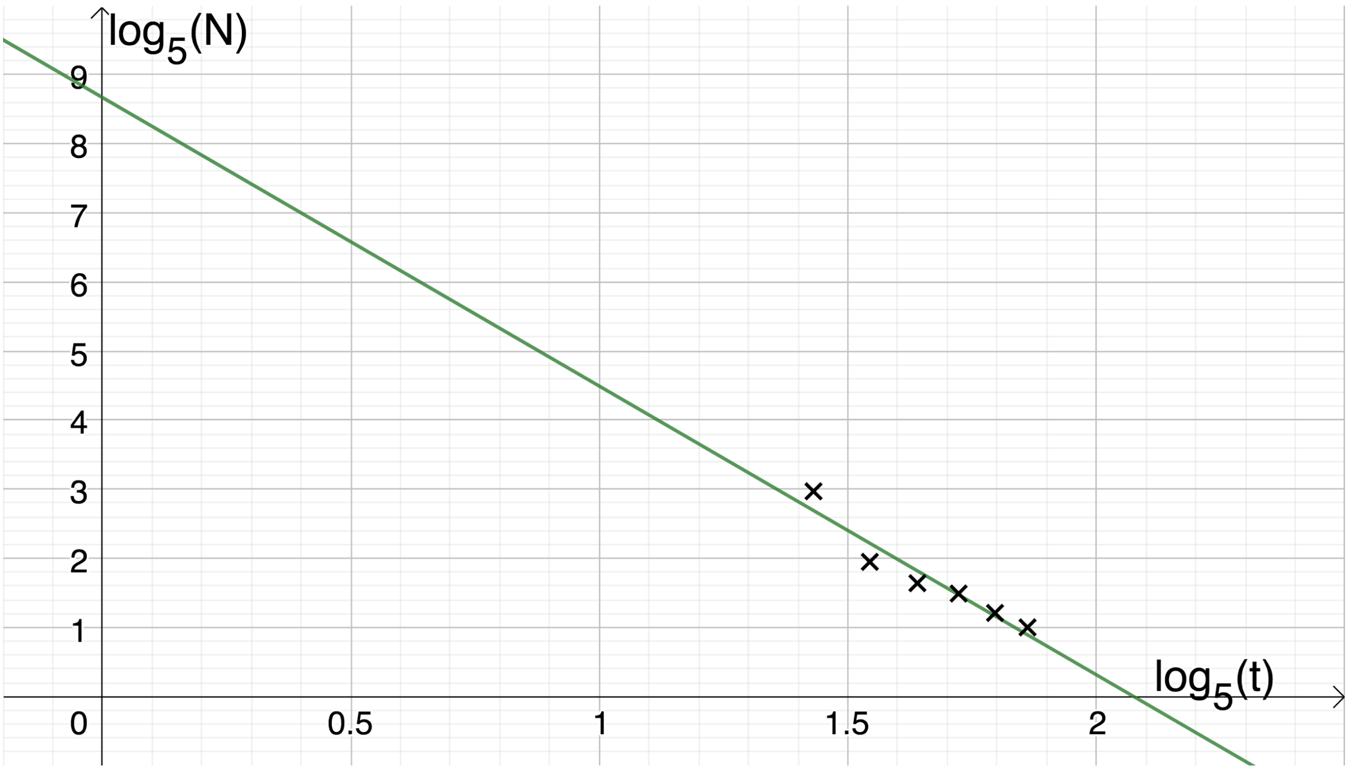
a. Show that, according to this model, the graph of $\log_5 N$ against $\log_5 t$ should be a straight line of gradient b. State, in terms of a, the intercept on the vertical axis.

$$N = at^b \Rightarrow \log_5 N = \log_5(at^b)$$
$$\Rightarrow \log_5 N = \log_5 a + \log_5 t^b$$
$$\Rightarrow \log_5 N = \log_5 a + b\log_5 t$$

Intercept on the vertical axis is $\log_5 a$

b. Complete the table of values below, writing your answers to 2 decimal places, and plot $\log_5 N$ against $\log_5 t$. Draw by eye a line of best fit for the data.

t	10	12	14	16	18	20
N	119	23	14	11	7	5
$\log_5 t$	1.43068	1.54396	1.63974	1.72271	1.79589	1.86135
$\log_5 N$	2.96944	1.94819	1.63974	1.4899	1.20906	1



c. Use your graph to find the equation for N in terms of t.

$$N = 1146428 \times t^{-4.178}$$

d. Use your results to estimate how many subscriptions there were to the magazine on January 1st 2000. Explain whether this is likely to be a correct figure.

$$N|_{t=1} = 1146428 \times 1^{-4.178} = 1146428$$

The model predicts that there were 1,146,428,000 subscriptions. Given that there are approximately 65 million people living in the UK, this figure is very high and unlikely to be correct.

9

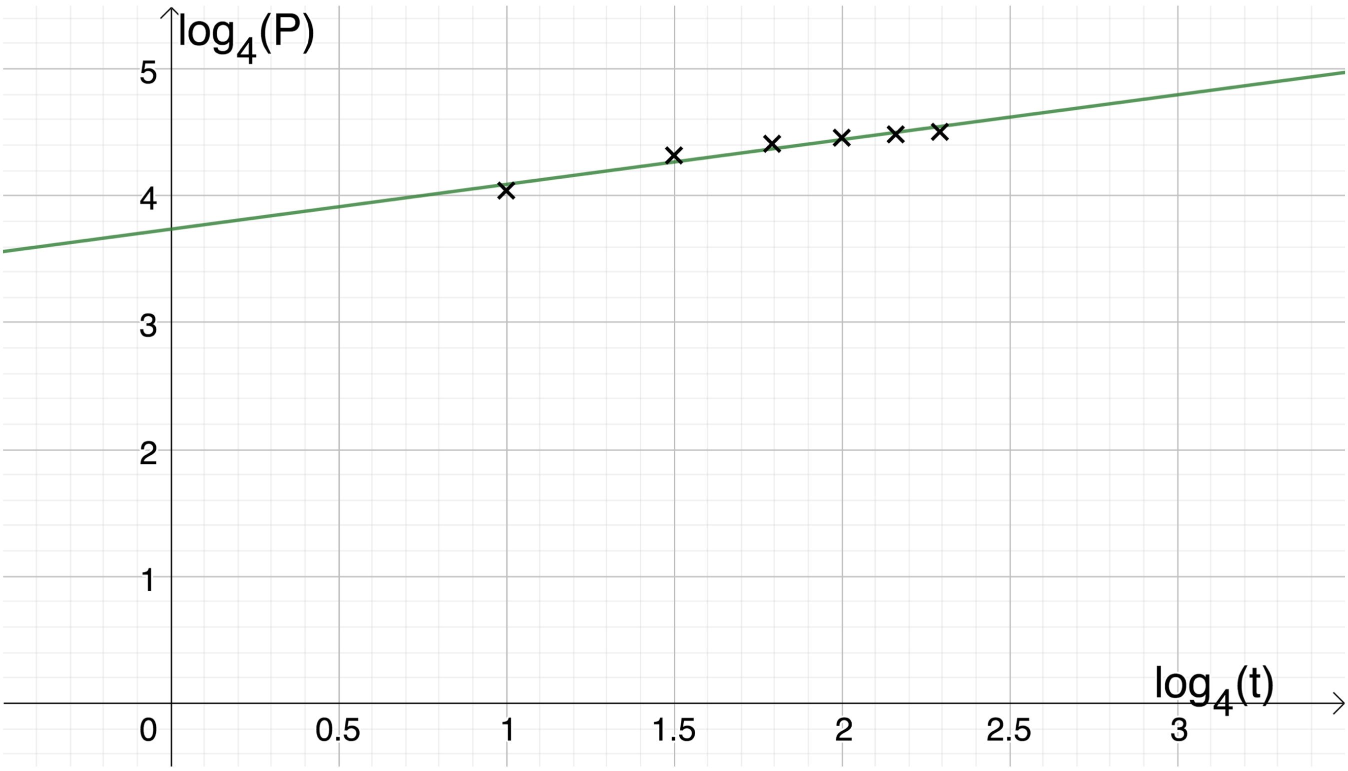
a. Show that, according to this model, the graph of $\log_4 P$ against $\log_4 t$ should be a straight line of gradient b. State, in terms of a, the intercept on the vertical axis.

$$P = at^b \Rightarrow \log_4 P = \log_4(at^b)$$
$$\Rightarrow \log_5 P = \log_4 a + \log_4 t^b$$
$$\Rightarrow \log_5 P = \log_4 a + b\log_4 t$$

Intercept on the vertical axis is $\log_4 a$

b. Complete the table of values below, writing your answers to 2 decimal places, and plot $\log_4 P$ against $\log_4 t$. Draw by eye a line of best fit for the data.

t	4	8	12	16	20	24
N	271	398	452	483	501	515
$\log_4 t$	1	1.5	1.79248	2	2.16096	2.29248
$\log_4 P$	4.04107	4.31831	4.41009	4.45794	4.48433	4.50421



c. Use your graph to find the equation for N in terms of t.

$$P = 177.813 \times t^{0.353}$$

10

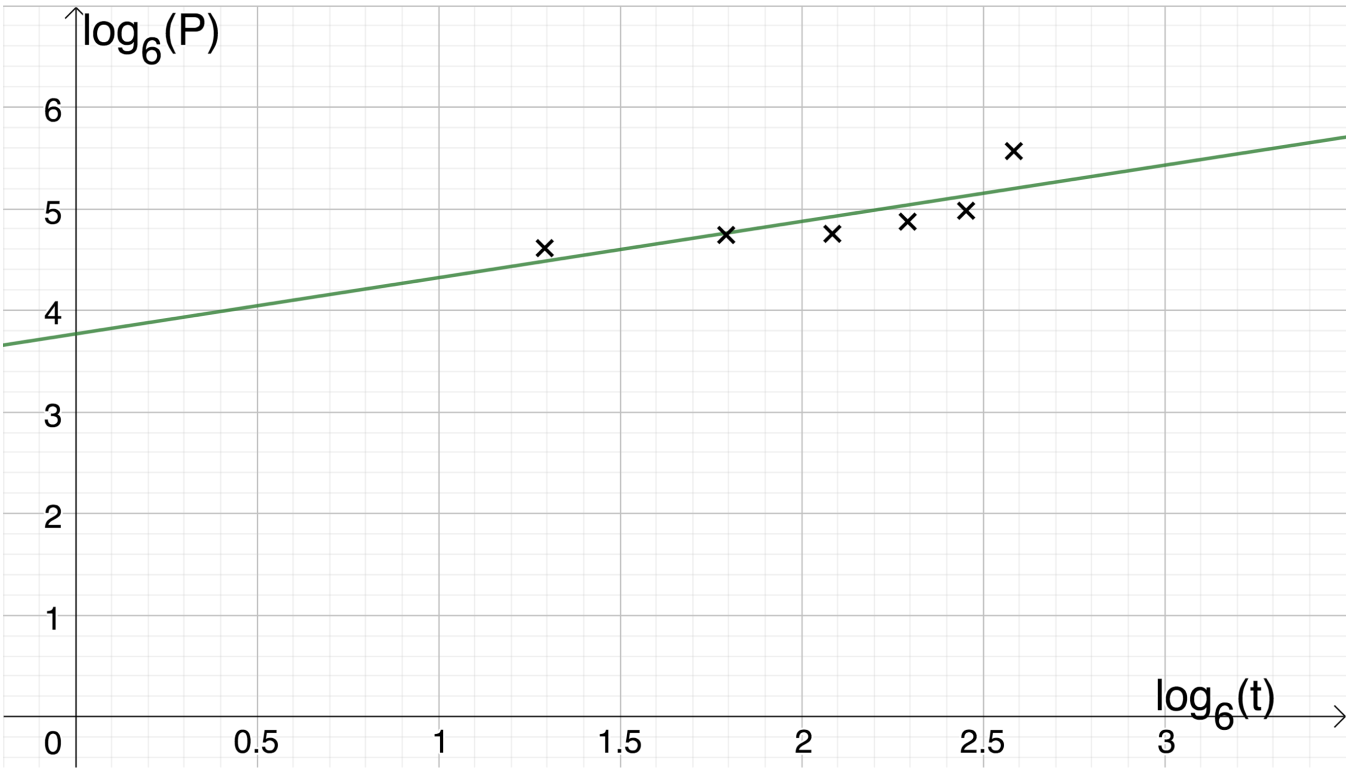
a. Show that, according to this model, the graph of $\log_6 P$ against $\log_6 t$ should be a straight line of gradient b. State, in terms of a, the intercept on the vertical axis.

$$P = at^b \Rightarrow \log_6 P = \log_6(at^b)$$
$$\Rightarrow \log_6 P = \log_6 a + \log_6 t^b$$
$$\Rightarrow \log_6 P = \log_6 a + b\log_6 t$$

Intercept on the vertical axis is $\log_6 a$

b. Complete the table of values below, writing your answers to 2 decimal places, and plot $\log_6 P$ against $\log_6 t$. Draw by eye a line of best fit for the data.

t	6	12	18	24	30	36
N	598.19	715.59	727.14	858.63	996.33	2248.15
$\log_6 t$	1.292481	1.792481	2.084963	2.292481	2.453445	2.584963
$\log_6 P$	4.61223	4.741495	4.753045	4.872946	4.98024	5.567261



c. Use your graph to find the equation for P in terms of t.

$$P = 855.591 \times t^{0.553}$$

d. Use your answers to predict the price of gold in January 1982. The actual price of gold was \$1050.49. Find the percentage difference between your prediction and the actual value.

$$P|_{t=60} = 855.91 \times 60^{0.553} \approx \$8236.56$$

Percentage difference

$$= \frac{8236.56 - 1050.49}{1050.49} \times 100 \approx 684\%$$